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The kinetic theory of heat and mass transfer from a spherical particle in a rarefied gas

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Abstract—Based on the solution of the gas-kinetic equation, the linear theory of heat and mass transfer processes in aerosol particle evaporation is formed. The dependence of heat and mass fluxes on the Knudsen number, evaporation–condensation and energy accommodation coefficients is investigated.

INTRODUCTION

INVESTIGATION of heat and mass transfer processes in a rarefied gas perturbed by a macroparticle suspended in it is the subject of aerosol microphysics [1].

Theories of this problem are usually reduced to the Knudsen gas model or hydrodynamic approximation. The present paper suggests the kinetic theory of heat and mass transfer processes in the neighbourhood of a spherical aerosol particle at arbitrary Knudsen numbers (*Kn*).

Heat transfer from a spherical particle into a quiescent gas at arbitrary *Kn* seems to have been first studied by Lees [2]. The solution of the kinetic equation in the four-moment approximation gave a simple analytical formula. Later this problem was tackled by Cercignani and Pagani [3] using the variational procedure based on the BGK linearized model equation.

Evaporation (or condensational growth) of an aerosol particle in a free-molecular regime ($Kn \gg 1$) is defined by the Hertz–Knudsen formula.

The boundary conditions for hydrodynamical equations [4] being represented by the expressions for temperature and density jumps [4–9], the linear approximation for the flux density of evaporated molecules with $Kn \to 0$ yields

$$J = a\frac{n_\infty}{2}\left(\frac{2kT_\infty}{\pi m}\right)^{1/2}(v_s - v_0 + \tau_s) \tag{1}$$

$$v_s = \frac{n_s - n_\infty}{n_\infty}, \quad v_0 = \frac{n'_\infty - n_\infty}{n_\infty}, \quad \tau_s = \frac{T_s - T_\infty}{T_\infty}.$$

The values of constant a taken from refs. [4–9] are given in Table 1.

Isothermal evaporation of a droplet at intermediate *Kn* seems to have been first studied by Lees [2] on the basis of the four-moment method used for solving the Boltzmann kinetic equation. The same method was applied in refs. [10, 11] where account was taken of the process nonisothermicity due to the evaporation-induced removal of the phase change heat.

The results obtained in refs. [2, 10, 11] for $Kn \to \infty$ correspond to the Hertz–Knudsen formula, whereas for $Kn \to 0$ they correspond to formula (1) with coefficient a which at $\alpha_m = 1$ is about twice as small as the values listed in Table 1.

In ref. [12] the problem was solved numerically based on the S-model kinetic equation for several intermediate *Kn* at $\alpha_m = 1$. The vapour density and temperature profiles near a droplet evaporating into vacuum were calculated.

The solution of the problem for the entire range of *Kn* was obtained in ref. [13]. The nonisothermicity of the evaporation process was assumed to be conditioned by the phase change heat. In the present paper, the parameters which characterize the relative supersaturation v_0 and the relative temperature difference τ_s are considered to be independent.

STATEMENT OF THE PROBLEM

It will be assumed that the relative supersaturation and the difference between particle and vapour temperatures are small, i.e. $|v_0| \ll 1$ and $|\tau_s| \ll 1$. In this case the state of the vapour is close to equilibrium and can be described by the distribution function which differs little from the Maxwellian function

$$f(\mathbf{r}, \mathbf{v}) = f_\infty[1 + v_0 + h(\mathbf{r}, \mathbf{v})] \tag{2}$$

where $\mathbf{r}$ is the radius vector originating from the particle centre; the equilibrium values of the temperature T_∞ and of the saturated vapour density $n_\infty(T_\infty)$ are the parameters of the Maxwellian function f_∞.

Assume that the fraction α_m of the molecules emitted from the particle surface is evaporated in an equilibrium manner, and the fraction $(1-\alpha_m)$ is reflected without having been condensed on collision with the particle. The velocities of the reflected molecules are distributed according to Maxwell's law with the temperature T_r, which generally differs from the surface temperature T_s, and with the corresponding number density n_r.

It will be assumed for the weakly non-equilibrium state that

NOMENCLATURE

E^{r}	energy flux of reflected molecules
E^{w}	energy flux of molecules reflected in the case of perfect accommodation
E^{-}	energy flux of molecules impacting on a surface
f	velocity distribution function of molecules
L	phase change heat per molecule
$N^{+}, N^{\mathrm{s}}, N^{-}$	fluxes of molecules emitted, evaporated in equilibrium manner and falling on the surface
n_{r}	density of molecules reflected from particle surface
$n_{\mathrm{s}}, n_{\infty}$	densities of saturated vapour molecules at temperatures T_{s} and T_{∞}
n'_{∞}	actual density of molecules far from particle
q	heat flux density
R_0	particle radius
T_{r}	temperature of molecules reflected from particle surface
$T_{\mathrm{s}}, T_{\infty}$	temperature of a particle and gas, respectively
U	macroscopic gas velocity.
Greek symbols	
α_{m}	evaporation–condensation coefficient
α_{E}	energy accommodation coefficient
η	gas viscosity coefficient.

$$T_{\mathrm{r}} = T_{\infty}(1+\tau_{\mathrm{r}}), \quad n_{\mathrm{r}} = n_{\infty}(1+\nu_{\mathrm{r}}),$$
$$|\nu_{\mathrm{r}}|, \quad |\tau_{\mathrm{r}}| \ll 1. \tag{3}$$

Then the boundary condition for the perturbation function will have the form

$$h(\mathbf{r}_0, \mathbf{v}) = \nu^{+} (c^2 - \tfrac{3}{2}) \cdot \tau^{+}$$
$$\mathbf{c} = \left(\frac{m}{2kT_{\infty}}\right)^{1/2} \mathbf{v}$$
$$(\mathbf{nv}) > 0$$
$$\nu^{+} = \alpha_{\mathrm{m}} \nu_{\mathrm{s}} + (1-\alpha_{\mathrm{m}}) \nu_{\mathrm{r}} - \nu_0$$
$$\tau^{+} = \alpha_{\mathrm{m}} \cdot \tau_{\mathrm{s}} + (1-\alpha_{\mathrm{m}}) \cdot \tau_{\mathrm{r}} \tag{4}$$

where $\mathbf{r}_0$ is the radius vector of the particle surface points.

Unknown reflection parameters ν_{r} and τ_{r} are determined from the balance equations for the number of molecules and energy on the particle surface

$$|N^{+}| = \alpha_{\mathrm{m}} \cdot |N^{\mathrm{s}}| + (1-\alpha_{\mathrm{m}}) \cdot |N^{-}|$$
$$|E^{\mathrm{r}}| = \alpha_{\mathrm{m}} \cdot |E^{\mathrm{w}}| + (1-\alpha_{\mathrm{E}})(1-\alpha_{\mathrm{m}}) |E^{-}|. \tag{5}$$

The equilibrium particle, T_{s}, and vapour, T_{∞}, temperatures are related to the corresponding molecular densities n_{s} and n_{∞} by the Clausius–Clapeyron linearized equation [14]

$$\nu_{\mathrm{s}} = (\beta - 1) \cdot \tau_{\mathrm{s}}, \quad \beta = \frac{L}{kT_{\infty}}. \tag{6}$$

Upon linearization and nondimensionalization, the kinetic equation with the collision integral in the form of the S-model [15] takes on the following form:

$$\mathbf{c}\frac{\partial h}{\partial \mathbf{r}} = -h + \nu + 2c_{\mathrm{r}} \cdot u + (c - \tfrac{3}{2}) \cdot \tau + \tfrac{4}{15} c_{\mathrm{r}} (c^2 - \tfrac{5}{2}) \cdot S \tag{7}$$

where

$$\nu = \frac{n - n_{\infty}}{n_{\infty}} - \nu_0 = \pi^{-3/2} \int \exp(-c^2) \cdot h \, d\mathbf{c}$$
$$u = \left(\frac{m}{2kT_{\infty}}\right)^{1/2} \cdot U = \pi^{-3/2} \int c_{\mathrm{r}} \cdot \exp(-c^2) h \, d\mathbf{c}$$
$$\tau = \frac{T - T_{\infty}}{T_{\infty}} = \pi^{-3/2} \int (\tfrac{2}{3} c^2 - 1) \exp(-c^2) h \, d\mathbf{c}$$
$$S = p_{\infty}^{-1} \left(\frac{m}{2kT_{\infty}}\right)^{1/2} \cdot q$$
$$= \pi^{-3/2} \int c_{\mathrm{r}} (c^2 - \tfrac{5}{2}) \exp(-c^2) h \, d\mathbf{c} \tag{8}$$

the dimensionless radius vector $\mathbf{r}$ is related to $(2kT_{\infty}/m)^{1/2} \cdot \eta / p_{\infty}$, which is equal to $2l/\sqrt{\pi}$ at $\eta = 0.5\, p_{\infty} (8kT_{\infty}/\pi m)^{1/2} \cdot l$; the dimensionless particle radius is related to Kn in the following way:

$$R = \left(\frac{m}{2kT_{\infty}}\right)^{1/2} \cdot \frac{p_{\infty}}{\eta} \cdot R_0 = \frac{\sqrt{\pi}}{2} Kn^{-1}. \tag{9}$$

Table 1. References [4–9]. The values of constant a in formula (1)

	References				
	[4]	[5]	[7]	[6]	[8, 9]
a	1.66	1.79	$\alpha_{\mathrm{m}}/(1-0.4\,\alpha_{\mathrm{m}})$ $\alpha_{\mathrm{m}} = 1$ 1.67	$\alpha_{\mathrm{m}}/(1-0.411\,\alpha_{\mathrm{m}})$ $\alpha_{\mathrm{m}} = 1$ 1.70	$\alpha_{\mathrm{m}}/(1-0.4004\,\alpha_{\mathrm{m}})$ $\alpha_{\mathrm{m}} = 1$ 1.67

Owing to the spherical symmetry of the problem, all macroscopic quantities depend only on the radial coordinate r.

THE SYSTEM OF INTEGRAL-MOMENT EQUATIONS

Kinetic equation (7), with boundary condition (4) taken into account, will be integrated along the selected direction of the velocity of molecules $\mathbf{\Omega} = \mathbf{c}/c$ [16]. The substitution of the resulting expression for the perturbation function into equation (8), which determines the macroparameters, will give the following integral equations for the vapour density and temperature profiles:

$$v = \pi^{-3/2}\int_{\omega}[v^{+}I_2+\tau^{+}(I_4-\tfrac{3}{2}I_2)]\cdot \mathrm{d}\omega$$

$$+\pi^{-3/2}\int_{V}[vI_1+2u\Omega_{r'}I_2+\tau(I_3-\tfrac{3}{2}I_1)$$

$$+\tfrac{4}{15}S\Omega_{r'}\cdot(I_4-\tfrac{5}{2}I_2)]\frac{\mathrm{d}\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^2}$$

$$\tau = \tfrac{2}{3}\pi^{-3/2}\int_{\omega}[v^{+}(I_4-\tfrac{3}{2}I_2)+\tau^{+}(I_6-3I_4+\tfrac{9}{4}I_2)]\,\mathrm{d}\omega$$

$$+\tfrac{2}{3}\pi^{-3/2}\int_{V}[v(I_3-\tfrac{3}{2}I_1)$$

$$+2u\Omega_{r'}\cdot(I_4-\tfrac{3}{2}I_2)+\tau(I_5-3I_3+\tfrac{9}{4}I_1)$$

$$+\tfrac{4}{15}S\Omega_{r'}(I_6-4I_4+\tfrac{15}{4}I_2)]\frac{\mathrm{d}\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^2}$$

$$\Omega_{r'} = \mathbf{\Omega}\cdot\frac{\mathbf{r}}{r'} = \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}\cdot\frac{\mathbf{r}'}{r'}$$

$$I_n(t) = \int_0^{\infty} x^n \exp(-x^2-t/x)\,\mathrm{d}x \qquad (10)$$

where ω is the integration domain representing a solid angle which the particle subtends at the fixed point with the radius vector $\mathbf{r}$; V the integration domain any point of which can be connected with the point $\mathbf{r}$ by a straight line, which does not intersect the particle; when integrating over the solid angle, the argument in the functions I_n is $t = |\mathbf{r}-\mathbf{r}_0|$ and when integrating over the domain, it is $V-t = |\mathbf{r}-\mathbf{r}'|$.

To obtain integral equations for the vapour and heat flux velocities use can be made of the continuity and energy conservation equations

$$\operatorname{div}\mathbf{u} = 0, \quad \operatorname{div}\mathbf{S} = 0.$$

From this, taking into account the spherical symmetry of the problem, the relationship can be easily established between the local values of the fluxes u and S at each point and their values u_R, S_R on the particle surface

$$u = \left(\frac{R}{r}\right)^2\cdot u_R, \quad S = \left(\frac{R}{r}\right)^2\cdot S_R. \qquad (11)$$

The quantities u_R and S_R depend on Kn, the evaporation–condensation coefficient α_{m}, the energy accommodation coefficient α_{E} and on the parameter β, which characterizes the phase change heat.

The integral equations for the local values of the fluxes u, S can be easily obtained from expressions (8) with the aid of the integral form of kinetic equation (7). Setting $r = R$ in these equations and integrating over the entire space V_0 about the particle, it is possible to obtain

$$u_R = \frac{1}{2\sqrt{\pi}}(v^{+}+\tfrac{1}{2}\tau^{+})-(\pi^{5/2}4R^2)^{-1}\int_{V_0}\mathrm{d}\mathbf{r}$$

$$\times\int_{\omega}\mathrm{d}\omega[vI_2-2u\cdot\Omega_{0r}\cdot I_3+\tau(I_4-\tfrac{3}{2}I_2)$$

$$-\tfrac{4}{15}S\Omega_{0r}(I_5-\tfrac{5}{2}I_3)] \qquad (12)$$

$$S_R = -\frac{1}{4\sqrt{\pi}}(v^{+}-\tfrac{7}{2}\tau^{+})-(4\pi^{5/2}\cdot R^2)^{-1}\int_{V_0}\mathrm{d}\mathbf{r}$$

$$\times\int_{\omega}\mathrm{d}\omega[v(I_4-\tfrac{5}{2}I_2)-2u\Omega_{0r}(I_5-\tfrac{5}{2}I_3)$$

$$+\tau(I_6-4I_4+\tfrac{15}{4}I_2)-\tfrac{4}{15}S\cdot\Omega_{0r}(I_7-5I_5+\tfrac{25}{4}I_3)]$$

$$\Omega_{0r} = \frac{\mathbf{r}-\mathbf{r}_0}{|\mathbf{r}-\mathbf{r}_0|}\cdot\frac{\mathbf{r}}{r}. \qquad (13)$$

The argument in $I_n(t)$ is $t = |\mathbf{r}-\mathbf{r}_0|$.

The system of equations (10)–(13) is incomplete, since it includes the unknown reflection parameters v_{r} and τ_{r}. These parameters are determined from balance equations (5) which are transformed into the following form:

$$\alpha_{\mathrm{m}}[v_{\mathrm{r}}-v_{\mathrm{s}}+\tfrac{1}{2}(\tau_{\mathrm{r}}-\tau_{\mathrm{s}})] = -2\sqrt{\pi}u_R$$

$$\alpha_{\mathrm{m}}(1-\alpha_{\mathrm{E}})(v_{\mathrm{r}}-v_{\mathrm{s}})-[\tfrac{7}{2}\alpha_{\mathrm{m}}(1-\alpha_{\mathrm{E}})+4\alpha_{\mathrm{E}}](\tau_{\mathrm{r}}-\tau_{\mathrm{s}})$$

$$= 4\sqrt{\pi}(1-\alpha_{\mathrm{E}})\cdot S_R. \qquad (14)$$

Provided the parameters v_0 and τ_{s} are independent, the system of equations (10)–(14), together with Clausius–Clapeyron equation (6), is closed. When the quantity τ_{s} is due to the phase change heat, it is determined from the condition of continuous radial heat flux on the particle surface [13].

SOLUTION OF INTEGRAL EQUATIONS

The greatest difficulty here is posed by the determination of the vapour density and temperature profiles, since it requires the solution of integral equations (10). If the functions $v(r)$ and $\tau(r)$ are known, the evaporation rate and heat flux are determined from equations (12) to (14).

It is necessary to employ the Bubnov–Galerkin method [16]. Equations (10) are constructed in such a way that the first terms on their right-hand sides describe the density and temperature profiles for $Kn \gg 1$. Therefore, rather accurate results can be expected for the evaporation rate and heat flux at all Kn, if approximations of the sought-after functions $v(r)$ and $\tau(r)$ correspond to the vapour density and temperature profiles in the continuous medium regime when $Kn \ll 1$.

The trial functions will be selected in the following form:

$$\tilde{v} = \frac{a}{r}, \quad \tilde{\tau} = \frac{b}{r}. \tag{15}$$

Constants a and b will be selected so that functions (15) could satisfy equations (10) in the best way. For this purpose, approximate functions (15) will be substituted into equations (10), and the requirement that the expressions obtained should be orthogonal with respect to r^{-1} will be imposed. The orthogonality condition for two arbitrary functions f_1 and f_2 has the form

$$\int_{V_0} f_1(r) \cdot f_2(r) \, \mathrm{d}\mathbf{r} = 0. \tag{16}$$

With equations (11)–(15) taken into account, the following system of algebraic equations is obtained to determine the unknown constants a, u_R, b, s_R, v_r and τ_r:

$$\sum_{j=1}^{6} \alpha_{ij} \cdot x_j = \beta_{i1} \cdot v_s + \beta_{i2} \cdot v_0 + \beta_{i3} \cdot \tau_s$$

$$(x_j) = a, u_R, b, s_R, v_r, \tau_r; \quad i = 1, \ldots, 6. \tag{17}$$

Coefficients α_{ij}, β_{ik} are cumbersome in form and will not be presented here.

It follows from equations (17) that

$$G_R = 2u_R = G_v(v_s - v_0) + G_\tau \cdot \tau_s \tag{18}$$

$$Q_R = \tfrac{4}{15} S_R = Q_v \cdot (v_s - v_0) + Q_\tau \cdot \tau_s \tag{19}$$

where the quantities G_v, G_τ, Q_v, Q_τ depend only on Kn, the evaporation–condensation coefficient α_m and on the energy accommodation coefficient α_E. Generally, the dependence of this kind is complex and cannot be represented analytically. This is possible only in the limiting cases of free-molecular ($R \to 0$) and almost continual ($R \gg 1$) regimes.

When $R \to 0$ ($\alpha_m = \alpha_E = 1$)

$$G_v = 2G_\tau = \frac{1}{\sqrt{\pi}}, \quad Q_v = -\tfrac{2}{7} Q_\tau = -(15\sqrt{\pi})^{-1}. \tag{20}$$

When $R \gg 1$ ($\alpha_m = \alpha_E = 1$), the treatment of numerical results by the least square method yields

$$G_v = 0.941 - 0.937R^{-1} + 0.517R^{-2} + \ldots$$

$$G_\tau = 0.941 - 1.731R^{-1} + 1.19R^{-2} + \ldots$$

$$Q_v = -0.105R^{-1} + 0.0903R^{-2} + \ldots$$

$$Q_\tau = 0.395R^{-1} - 0.72R^{-2} + \ldots \tag{21}$$

It should be noted, that equations (20) correspond to the Hertz–Knudsen formula, whereas equations (21) for G_v and G_τ show a satisfactory agreement with the result of ref. [4].

For the intermediate values of the rarefaction parameter R, numerical computation was made, the results of which are listed in Tables 2–4. The computational accuracy is almost completely determined by the accuracy with which the Galerkin coefficients α_{ij} were calculated. The error does not exceed 0.2% at any R.

An indirect verification of the method used to solve integral moment equations and also of the computational accuracy can be provided by the fulfillment of the Onsager reciprocity relations. Considering the particle and the surrounding vapour as a closed two-phase system, it is possible to obtain the equation for entropy production of the form [17]

$$\sigma_s = (q + kT_\infty \cdot J) \cdot x_1 + J \cdot x_2 \tag{22}$$

where

Table 2. The values of G_v depending on the rarefaction parameter R and evaporation coefficient $\alpha_m(\alpha_E = 1)$

				α_m			
R	0.1	0.2	0.4	0.5	0.7	0.8	1.0
0.01	0.0565	0.113	0.226	0.283	0.396	0.453	0.567
0.05	0.0565	0.113	0.228	0.285	0.401	0.459	0.576
0.1	0.0566	0.114	0.229	0.287	0.405	0.465	0.585
0.3	0.0569	0.115	0.233	0.294	0.418	0.481	0.612
0.5	0.0570	0.115	0.236	0.298	0.427	0.494	0.633
0.7	0.0572	0.116	0.238	0.302	0.435	0.505	0.650
1.0	0.0573	0.117	0.241	0.307	0.445	0.518	0.672
2.0	0.0577	0.118	0.248	0.317	0.467	0.548	0.724
5.0	0.0582	0.120	0.256	0.331	0.499	0.592	0.803
7.0	0.0583	0.121	0.259	0.336	0.509	0.607	0.830
10.0	0.0584	0.121	0.261	0.340	0.517	0.619	0.582
20.0	0.0586	0.122	0.265	0.346	0.533	0.641	0.895
100.0	0.0587	0.123	0.268	0.351	0.546	0.659	0.932

Table 3. The values of G_τ depending on the rarefaction parameter R and evaporation coefficient $\alpha_m(\alpha_E = 1)$

				α_m			
R	0.1	0.2	0.4	0.5	0.7	0.8	1.0
0.01	0.0282	0.0565	0.113	0.141	0.198	0.227	0.283
0.05	0.0284	0.0568	0.114	0.143	0.201	0.230	0.289
0.1	0.0287	0.0575	0.116	0.145	0.205	0.235	0.296
0.3	0.0301	0.0607	0.123	0.155	0.221	0.255	0.324
0.5	0.0316	0.0639	0.131	0.165	0.237	0.274	0.351
0.7	0.0331	0.0671	0.138	0.175	0.252	0.292	0.376
1.0	0.0352	0.0715	0.148	0.188	0.273	0.318	0.412
2.0	0.0407	0.0833	0.175	0.224	0.330	0.387	0.511
5.0	0.0489	0.101	0.216	0.279	0.420	0.499	0.676
7.0	0.0514	0.106	0.228	0.296	0.449	0.535	0.732
10.0	0.0535	0.111	0.239	0.311	0.474	0.566	0.780
20.0	0.0561	0.117	0.254	0.331	0.510	0.614	0.857
100.0	0.0582	0.122	0.266	0.348	0.541	0.654	0.924

$$x_1 = \tau_s/T_\infty, \quad x_2 = k(v_s - v_0)$$

$$q + kT_\infty \cdot J = L_{11} \cdot x_1 + L_{12} \cdot x_2$$

$$J = L_{21} \cdot x_1 + L_{12} \cdot x_2.$$

According to the reciprocity principle, the cross kinetic coefficients should be equal; i.e. $L_{12} = L_{21}$ or, with equations (18) and (19) taken into account

$$G_\tau = \tfrac{15}{2} Q_v + G_v. \tag{23}$$

It is obvious from the computation, that miscalculation of equation (23) does not exceed 0.2% at all values of *Kn*, the evaporation–condensation coefficient α_m and the energy accommodation coefficient α_E.

DISCUSSION OF RESULTS

The quantity G_v characterizes an isothermal vapour flow to or from the particle depending on whether v_0 is positive or negative.

The quantity G_τ characterizes a non-isothermal vapour flow induced by the difference in the thermal velocities of molecules near and at a distance from a particle, toward lower temperatures.

The non-isothermal heat flux Q_τ is conditioned, along with 'conventional' mechanisms (vapour thermal conductivity and Knudsen heat transfer), also by the energy transfer together with the non-isothermal vapour flow. The calculation shows that the contribution of the latter mechanism can amount to about 21% in the continual regime and about 13% in the free-molecular regime.

The isothermal heat flux characterized by the quantity Q_v is conditioned by the difference between the mean energy transferred by the isothermal vapour flow and the enthalpy flow [17].

The isothermal and non-isothermal components of the vapour flow increases monotonically with a decrease of the parameter R (from the continuum to the free-molecular regime). The decrease of the quantities G_v and G_τ with R should be clarified. The point here is that a decrease in the rarefaction parameter R at a fixed particle temperature does not effect the flow density of molecules evaporating in an equilibrium manner, but it increases the density of the

Table 4. The values of Q_τ depending on the rarefaction parameter R and evaporation coefficient $\alpha_m(\alpha_E = 1)$

				α_m			
R	0.1	0.2	0.4	0.5	0.7	0.8	1.0
0.01	0.149	0.147	0.143	0.141	0.137	0.136	0.132
0.05	0.149	0.147	0.143	0.141	0.137	0.135	0.131
0.1	0.148	0.146	0.142	0.140	0.137	0.135	0.131
0.3	0.144	0.142	0.139	0.137	0.132	0.130	0.126
0.5	0.140	0.138	0.134	0.132	0.127	0.125	0.121
0.7	0.135	0.133	0.129	0.127	0.122	0.120	0.115
1.0	0.127	0.125	0.121	0.119	0.115	0.113	0.108
2.0	0.105	0.104	0.100	0.0981	0.0939	0.0917	0.0868
5.0	0.0667	0.0656	0.0632	0.0619	0.0589	0.0572	0.0534
7.0	0.0531	0.0522	0.0503	0.0492	0.0468	0.0454	0.0423
10.0	0.0404	0.0398	0.0383	0.0375	0.0357	0.0346	0.0322
20.0	0.0223	0.0220	0.0212	0.0208	0.0198	0.0192	0.0178
100.0	0.00484	0.00477	0.00461	0.00451	0.00430	0.00417	0.00387

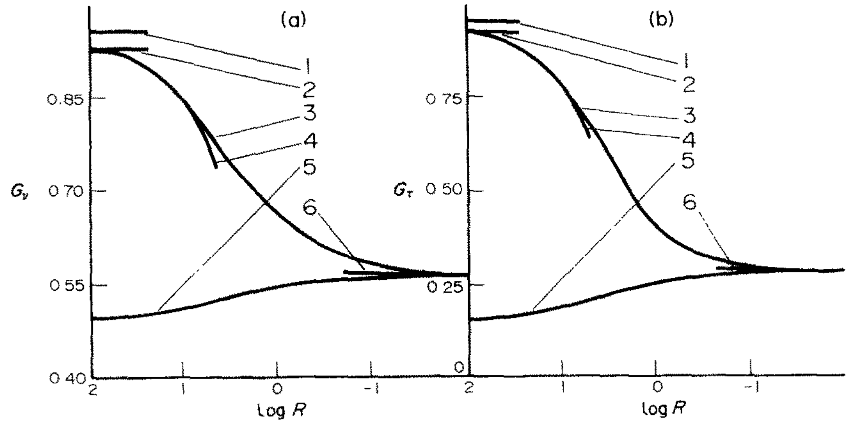

FIG. 1. Comparison of different theories for $\alpha_m = 1$: 1, Makashev [6]; 2, Labuntsov [7], Cipolla *et al.* [8], Chernyak and Suetin [9]; 3, the present study; 4, Sone and Onishi [4]; 5, Lees [2], Lou [11]; 6, Hertz–Knudsen formula.

flow of molecules impacting on the particle. As a result, the absolute magnitude of the overall vapour flow decreases. This decrease is a maximum at $\alpha_m = 1$; for G_v it decreases by a factor of ~ 1.7 and for G_τ by a factor of ~ 3.3.

It should be noted that unlike the evaporation rate (growth) and isothermal heat flux, the non-isothermal heat flux depends relatively weakly on the evaporation coefficient. A decrease in α_m from 1 to 0 leads to an increase of Q_τ by about 13% in the free-molecular regime and by about 21% in the continual regime.

Comparison of different theories of evaporation rate at $\alpha_m = 1$ is given in Fig. 1. It is seen that the result of the four-moment approximation [2, 10, 11] gives an incorrect dependence of the quantities G_v and G_τ on the rarefaction parameter R and, consequently, an incorrect hydrodynamic limit. Presumably, this can be attributed to the inadequacy of the four-moment approximation. When $R \gtrsim 10$, asymptotic equations (21) differ from the results presented in ref. [4] within 1% and when $R \approx 10$ they differ from numerical calculation by less than 5%.

Consider the state of dynamic equilibrium in which the processes of evaporation and condensation induced by the independent factors ν_0 and τ_s are mutually compensated. In this case, equation (18), together with the Clausius–Clapeyron equation (6), yields

$$K_p = G_v(\nu_0/\tau_s)[(\beta-1)\cdot G_v + G_\tau]^{-1} = 1. \quad (24)$$

When $K_p < 1$, the droplet evaporates, and when $K_p > 1$, condensation growth occurs.

Thus, if the parameters ν_0 and τ_s are independent, K_p is the criterion for determining the resulting direction of the phase change process.

Table 5 lists the values of the quantity $q^* = Q_\tau/Q_{\tau K}$, where $Q_{\tau K} = 4\alpha_E(15\sqrt{\pi})^{-1}$, depends on the rarefaction parameter R and the energy accommodation coefficient α_E in the absence of evaporation–condensation processes ($\alpha_m = 0$).

Figure 2 presents the comparison of theoretical results with the experimental data of Takao [18] for $\alpha_E = 1$. A certain difference in theories is observed only at intermediate values of parameter R.

Lees' results [2] systematically undershoot the

Table 5. The values of q^* depending on the rarefaction parameter R and energy accommodation coefficient $\alpha_E(\alpha_m = 0)$

R	α_E 0.1	0.3	0.5	0.7	0.9	1.0
0.05	1.000	1.000	1.000	1.000	1.000	1.000
0.1	1.000	0.999	0.999	0.998	0.998	0.997
0.3	0.997	0.991	0.986	0.980	0.975	0.972
0.5	0.994	0.981	0.969	0.957	0.946	0.940
0.7	0.990	0.970	0.951	0.932	0.915	0.906
1.0	0.984	0.952	0.923	0.895	0.869	0.856
2.0	0.961	0.890	0.831	0.778	0.731	0.710
3.0	0.937	0.833	0.749	0.681	0.624	0.599
4.0	0.914	0.780	0.680	0.603	0.541	0.515
5.0	0.891	0.732	0.621	0.539	0.476	0.430
7.0	0.848	0.650	0.527	0.443	0.383	0.358
10.0	0.789	0.555	0.428	0.349	0.294	0.272
20.0	0.639	0.371	0.261	0.202	0.164	0.150

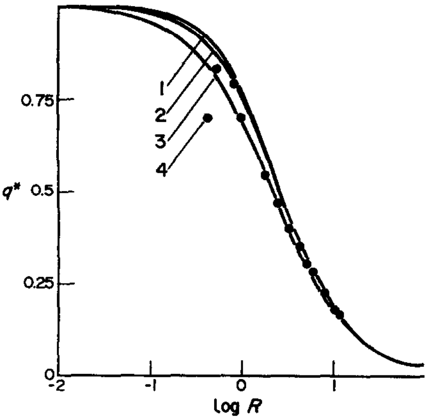

FIG. 2. The reduced heat flux q^* vs the rarefaction parameter R at $\alpha_E = 1$: 1, the present study; 2, Cercignani and Pagani [3]; 3, Lees [2]; 4, Takao's experiment [18].

values of q^* (discrepancy amounts to about 12%), presumably due to the approximation of the four-moment method. The maximum discrepancy between the results obtained in the present study for $\alpha_E = 1$ and those of the BGK model of ref. [3] comes to about 3%.

In ref. [18] the heat flux in air from a heated glass ball with a diameter of about 1.5 cm placed in a vessel with a characteristic size of about 1 m was measured. The minimum discrepancy between the experiment and the results of this publication amounts to about 5% at $R \approx 10$, while the maximum discrepancy comprises almost 14% at $r \approx 1$. Unfortunately, the experimental technique used did not permit measurements at lower pressures ($R < 1$), since the effect of the external volume became appreciable and the experimental points were widely scattered.

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LA THEORIE CINETIQUE DU TRANSFERT DE CHALEUR ET DE MASSE POUR UNE PARTICULE SPHERIQUE DANS UN GAZ RAREFIE

Résumé—A partir de la solution de l'équation de la cinétique des gaz, on formule la théorie linéaire des mécanismes du transfert de chaleur et de masse dans l'évaporation des particules d'aérosol. On étudie la dépendance des flux de chaleur et de masse vis-à-vis du nombre de Knudsen, des coefficients d'évaporation–condensation et d'accommodation d'énergie.

DIE KINETISCHE THEORIE DER WÄRME- UND STOFFÜBERTRAGUNG VON EINEM KUGELFÖRMIGEN TEILCHEN IN EINEM VERDÜNNTEN GAS

Zusammenfassung—Mit Hilfe der Lösung der gas-kinetischen Gleichung wird eine lineare Theorie der Vorgänge von Wärme- und Stoffübertragung bei der Verdunstung eines Aerosol-Teilchens formuliert. Die Abhängigkeit des Wärme- und Stoffstroms von der Knudsen-Zahl sowie von den Koeffizienten für Verdampfung, Kondensation und Akkommodation wird untersucht.

КИНЕТИЧЕСКАЯ ТЕОРИЯ ТЕПЛОМАССОПЕРЕНОСА В РАЗРЕЖЕННОМ ГАЗЕ ОТ СФЕРИЧЕСКОЙ ЧАСТИЦЫ

Аннотация—На основе решения газокинетического уравнения строится линейная теория процессов тепломассопереноса при испарении аэрозольной частицы. Исследуется зависимость потоков тепла и массы от числа Кнудсена, коэффициентов испарения–конденсации и аккомодации энергии.